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ASYMPTOTIC BEHAVIOR OF SINGULAR VALUES OF CONVOLUTION OPERATORS

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1. Introduction. In [1] a study was made of the singular values and singular functions of the convolution operator

$$\tilde{K} \cdot = \int_0^x K(x-y) \cdot dy, \quad 0 \leqq x \leqq 1, \tag{1.1}$$

under the condition that $K(u)$ is reasonably smooth and $K(0) \neq 0$. Asymptotic estimates of the singular functions and values were obtained. A somewhat heuristic argument was made to suggest that quite different behaviors are to be expected in the event that $K(0) = 0$.

In this paper we treat the case

$$K(u) = u^n k(u), \quad 0 \leqq u \leqq 1, \tag{1.2}$$

where n is a positive integer, $k(u) \in C^n [0, 1]$, and $k(0) \neq 0$. We are unable to obtain asymptotic estimates for the singular functions, but we do obtain such results for the singular values. This is done by showing that the singular values of $K(u)$ and those of $k(0)\, u^n$ differ little for large indices.

2. Some preliminaries. It is shown in [1] that instead of studying the nonsymmetric operator $\tilde{K}$ we may confine our attention to the symmetric operator

$$K \cdot = \int_{1-x}^1 K(x+y-1) \cdot dy, \; 0 \leqq x \leqq 1. \tag{2.1}$$

The singular values of $\tilde{K}$ are just the absolute values of the eigenvalues of K. It is also convenient to assume

$$k(0) = 1. \tag{2.2}$$

The "comparison operator" now becomes

$$K_n \cdot = \int_{1-x}^1 K_n(x+y-1) \cdot dy, \tag{2.3}$$

with

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