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A LIMIT ON THE LENGTH OF THE INDECOMPOSABLE MODULES OVER A HEREDITARY ALGEBRA

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Introduction. Let k be an algebraically closed field, let Λ be a hereditary k-algebra of finite type, that is, there is only a finite number of nonisomorphic indecomposable Λ-modules. Let K be an indecomposable Λ-module, let Soc K be the socle of K, $P(\mathrm{Soc}\ K)$ the projective cover of Soc K, and $I(K)$ the injective envelope of K. If X is a module, let $\ell(X)$ denote the composition length of X. In this paper we show that the inequality

$$\ell(P(\mathrm{Soc}\ K)) + \ell(I(K)) - \ell(K) \leqq \ell(M) \tag{1}$$

is always true, where M is an indecomposable Λ-module of maximal length.

It is known that if $p(\Lambda)$ is the number of nonempty preprojective classes in the preprojective partition for Λ, then $p(\Lambda) = \ell(M)$, where M is as above [5]. We will apply the inequality (1) above to prove that if R is the trivial extension of Λ by $D\Lambda$, then we have

$$p(R) - 1 \leqq \ell(X_0) \leqq p(R) \tag{2}$$

where X_0 is an indecomposable R-module of maximal length.

To show this, we first apply (1) to show that if X is an indecomposable R-module, then

$$\text{(3)} \quad \begin{aligned} &\text{(i) } \ell(X) \leqq 1 + \ell(M), \text{ if } X \text{ is projective} \\ &\text{(ii) } \ell(X) \leqq \ell(M), \text{ if } X \text{ is not projective.} \end{aligned}$$

Using (3) and the fact that $p(R) = p(\Lambda) + 1$, we get that (2) is always satisfied.

In the last section we give some examples to illustrate what may happen if the algebra is not hereditary. First we give an example of an algebra where the inequality (1) is not satisfied by all indecomposable modules, not even by all simple modules.

As a second example, we present an algebra Λ which is not hereditary, but the inequality (1) is satisfied for all indecomposable Λ-modules, and

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