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# HYPERBOLIC OPERATORS IN SPACES OF GENERALIZED DISTRIBUTIONS

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Hyperbolic operators were investigated by L. Ehrenpreis [**4**] in the space of Schwartz distributions and by C. C. Chou [**3**] in the spaces of Roumieu ultradistributions. In this paper we study hyperbolic operators in spaces of Beurling generalized distributions. (See [**1**] and [**2**]).

Let $D'$, $E'$, $D'_\omega$, $E'_\omega$ be the spaces of distribution, distributions with compact support, generalized distributions and generalized distributions with compact support in $\mathbf{R}^n$, respectively.

DEFINITION. The convolution operator $S$, $S \in E'_\omega$, is said to be $\omega$-hyperbolic with respect to $t > 0$ (resp. $t < 0$) if there exists a fundamental solution $E^+$ (resp. $E^-$), $E^+; E^- \in D'_\omega$, so that supp $E^+ \subset \{(x, t) \in \mathbf{R}^n \times \mathbf{R}: t \geqq -b_0 + b_1|x|\}$ for some $b_0, b_1 > 0$ (resp. supp $E^- \subset \{(x, t) \in \mathbf{R}^n \times \mathbf{R}: t \leqq b_0 - b_1|x|\}$ for some $b_0, b_1 > 0$).

An operator is said to be $\omega$-hyperbolic if it is $\omega$-hyperbolic with respect to $t > 0$ and $t < 0$. This definition coincides with the definition of hyperbolicity introduced by Ehrenpreis [**4**, Theorem 2] for Schwartz distributions.

For the notation and the properties of generalized distributions we refer to [**2**]. Let $\omega \in \mathcal{M}_c$ (see [**2**, Definition 1.3.23]). Using Proposition 1.2.1 of [**2**] we could extend $\omega$ to $\mathbf{C}^n$ without losing any of its original properties; we will assume that $\omega$ is the extended function. We use the estimate

$$(1) \qquad \omega(\xi) = o(|\xi/\log|\xi|), \text{ as } |\xi| \to \infty,$$

from which it follows that

$$(2) \qquad \omega(\xi) \leqq M(1 + |\xi|),$$

for some constant $M$.

Following Ehrenpreis we prove the following theorem which characterizes $\omega$-hyperbolic operators. The theorem and its proof will be given in the case of $\omega$-hyperbolicity with respect to $t > 0$, the other case could be proved similarly.

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