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STURM-LIOUVILLE DIFFERENTIAL OPERATORS IN DIRECT SUM SPACES

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ABSTRACT. Sturm-Liouville boundary value problems on two intervals are studied in the setting of the direct sum of the L^2 spaces of functions defined on each of the separate intervals. The interplay between these two L^2 spaces is of critical importance. This study is partly motivated by the occurrence of S-L problems with coefficients that have a singularity in the interior of the basic interval. Such problems are not uncommon in the applied mathematics and mathematical physics literature.

1. Introduction. Sturm-Liouville (S-L) problems with coefficients which have a singularity in the interior of the basic interval under consideration have recently been studied in the Physics literature [**2, 5**]. Here the interior singular point is viewed as a left end point of one interval and a right end point of another. In effect, then, we have two differential expressions: one for functions defined on interval I_1, the other for functions defined on I_2. For the general theory developed below whether the right end point of I_1 is the same as the left end point of I_2 is of no importance. Indeed the intervals I_1 and I_2 are to be taken as arbitrary; they may be disjoint, overlap, or even be identical and with the same or different differential expressions.

The purpose of this paper is to provide an operator theoretic framework for the study of two differential operators together: M_1 defined on an interval I_1 and M_2 defined on I_2. In particular we define a minimal and a maximal operator each associated with both expressions and characterize all self-adjoint extensions of the minimal operator in terms of "boundary conditions". These conditions involve both expressions on both intervals.

In the regular case they can be interpreted in terms of the values of the unknown function f and its quasi-derivative at all four end points. These conditions include the so called "interface" conditions obtained by other methods (see [**8**]). A special case of these interface conditions is the so called condition for a "point interaction of strength α". (see [**5**, pp. 20, 21]).

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