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BOUNDARY VALUE PROBLEMS WITH JUMPING NONLINEARITIES

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1. Introduction. Let Ω be a bounded domain in $\mathbf{R}^n$ with sufficiently smooth boundary and consider the boundary value problem

$$(1.1)\qquad \begin{aligned} \Delta u + f(u) &= g(x), \quad x \in \Omega \\ u &= 0, \qquad x \in \partial \Omega \end{aligned}$$

where $f \in C^1(\mathbf{R})$ and g is Holder continuous on $\bar{\Omega}$. Let $\lambda_0 < \lambda_1 \leqq \lambda_2 \leqq \cdots$ denote the set of eigenvalues of the homogeneous problem

$$(1.2)\qquad \begin{aligned} \Delta u + \lambda u &= 0, \qquad x \in \Omega \\ u &= 0, \qquad x \in \partial \Omega. \end{aligned}$$

Assuming that the limits

$$\alpha = \lim_{s \to -\infty} f'(s), \qquad \beta = \lim_{s \to \infty} f'(s)$$

exist and satisfy

$$\alpha < \lambda_0 < \beta < \lambda_1,$$

Ambrosetti and Prodi [3] showed that there exists in $C^{0,\alpha}(\bar{\Omega})$ a connected C^1 manifold M separating $C^{0,\alpha}(\bar{\Omega})$ into components A_1 and A_2 such that:

(i) if $g \in A_1$, then (1.1) has no solution;
(ii) if $g \in M$, then (1.1) has a unique solution; and
(iii) if $g \in A_2$, then (1.1) has exactly two solutions.

This fundamental paper has generated much interest, and many interesting generalizations, extensions and refinements have since appeared (see, e.g., Manes/Micheletti [22], Kazdan/Warner [18], Dancer [6, 7], Amann/Hess [2], Hess [15], Fucik [10–12], Lazer/McKenna [19–21], Hofer [16], Ruf [25], and Solimini [27]).

Rewriting g as $t\theta + h$, where θ is a positive eigensolution of (1.2) corresponding to λ_0, it was Dancer [7] who showed that if $\alpha < \lambda_0 < \beta$, then there exists t_0 such that:

(i) if $t < t_0$, (1.1) has no solution; and

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