ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 16, Number 3, Summer 1986

SOLID FUEL COMBUSTION-
SOME MATHEMATICAL PROBLEMS

J. W. BEBERNES

1. Introduction. The initiation of a combustion process involves a myriad of complex physical phenomena which are fascinating to observe and challenging to describe in quantitative terms. In general one is concerned with the time-history of a spatially varying process occurring in a deformable material in which there is a strong interaction between chemical heat release, diffusive effects associated with the transport properties, bulk material motion as well as several types of propagating wave phenomena. Mathematical models capable of describing these combustion systems incorporate not only familiar reaction-diffusion effects associated with rigid materials, but those arising from material compressibility as well. For a combustible gas, the complete reactive Navier-Stokes equations are required to describe the phenomena involved.

In this paper, we shall focus on the initiation and evolution of thermal explosion processes in rigid materials. In this situation the physical processes are determined by a pointwise balance between chemical heat addition and heat loss by conduction.

The mathematical system which describes a thermal reaction event for a gaseous fuel in a bounded container is given in §2. Also in this section, we show how the complete system (c) can be simplified for a rigid fuel to a reactive diffusive system (2.1)–(2.2), and to the ignition model (2.3)–(2.4) by activation energy asymptotics. Closely related to the ignition model are the steady-state problems (2.5)–(2.6) and (2.7)–(2.8), referred to here as the Gelfand problem [7] and the perturbed Gelfand problem, respectively.

In §3, we survey some known results for such steady state problems for rather general domains Ω. Then in §4 we give more precise multiplicity results for the case $\Omega = B_1$, a ball in $\mathbf{R}^n$. In §5, we study the solution profiles for these steady-state models and in §6 we return to the classical ignition model to analyze the problem of thermal runaway.

Research supported by the U.S. Army Research Office under contract no. DAAG 29–82–K–0069.