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OSCILLATORY AND ASYMPOTOTIC BEHAVIOR OF CERTAIN FOURTH ORDER DIFFERENCE EQUATIONS

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Introduction. In several recent papers the oscillatory and asymptotic behavior of solutions of second order difference equations have been discussed, e.g., [1], [2], [3]. However, when compared to differential equations, the study of the oscillation properties of difference equations has received little attention, especially for orders greater than two.

This note is concerned with the solutions of the fourth order linear difference equation

$$(1) \qquad \Delta(\Delta^3 u_n + p_n u_{n+2}) + p_n \Delta u_{n+1} + q_n u_{n+2} = 0$$

where Δ denotes the differencing operator, i.e., $\Delta x_n = x_{n+1} - x_n$. While no sign conditions are explicitly stated for the real sequence $\{p_n\}$, it will be assumed that $q_n > 0$ for each n.

By a solution of (1) we will mean a real sequence $\{u_n\}$ defined on the set of nonnegative integers which satisfies (1). A nontrivial solution of (1), say $\{u_n\}$, is called *nonoscillatory* if there exists $n_0 \geqq 0$ such that $u_m u_{m+1} > 0$ for all $m \geqq n_0$; otherwise it is said to be oscillatory.

The results established herein are extensions to difference equations of certain results obtained by Taylor in [6]. It is clear that (1) is a discrete analogue of the equation

$$(y''' + p(x)y)' + p(x)y' + q(x)y = 0.$$

Moreover, we shall show herein that certain techniques developed to study this differential equation can be used to great advantage in the study of (1).

Main Results. Let V denote the solution space of (1). To begin our study of (1) we consider the following operator defined on V: For each $\{u_n\} \in V$, define

$$(2) \qquad F_n = F[u_n] = u_{n+1}[\Delta^3 u_n + p_n u_{n+2}] - \Delta u_n \Delta^2 u_n.$$

Computing the difference of F_n and making appropriate substitutions we find that