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FREQUENCY OF COPRIMALITY OF THE VALUES OF A POLYNOMIAL AND A PRIME-INDEPENDENT MULTIPLICATIVE FUNCTION*

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1. Introduction. Let H, J, and n denote positive integers, take P to be a polynomial with integer coefficients, and assume that M is a nonzero integer-valued multiplicative function such that

$$M(p) = H, \qquad M(p^2) = J, \tag{1}$$

for every prime p. In 1976, E. J. Scourfield **[4]** obtained the estimate

$$\#\{n \leqq x: (M(n), n) = 1\} \sim c_M x, \tag{2}$$

as x tends to ∞. She obtained equally precise results for elements of the class of "polynomial-like" arithmetic functions—a class which includes ϕ and σ. Five years later, in the Ph.D. dissertation of the author **[6**, §3.4**]**, we obtained an estimate for the left side of (2) which is more precise, if a certain convergence condition is satisfied. For example, we showed that

$$\#\{n \leqq x: (d(n), n) = 1\} = c_d x + 0(\sqrt{x}(\log x)^3), \tag{3}$$

where $d(n)$ is the number of positive integers dividing n, and c_d is a computable constant with $0 < c_d < 1$. In this paper, we derive the following estimate for $\#\{n \leqq x: (M(n), P(n)) = 1\}$.

THEOREM 1. $\#\{n \leqq x: (P(n), M(n)) = 1\} = C_{M,P}x + 0(\sqrt{x}(\log x)^{2J} E(x, M))$, *where*

$$C_{M,P} = \frac{6}{\pi^2 H} \sum_{t=1}^{\infty} \frac{1}{tU} \prod_{p \mid tU} (1 - p^{-2})^{-1} \sum_{\substack{b \bmod U \\ (P(bt), U) = (b, U, t) = 1 \\ \mu((q, U)) \neq 0}} \prod_{\substack{p \mid t(b, U) \\ p \nmid U/(b, U)}} (1 - p^{-1}),$$

where $U = HM(t)$, *and*

$$E(x, M) = \sum_{\substack{t \leq x \\ t \text{ cubefull}}} 2^{\omega(t)} |M(t)| t^{-1/2}.$$

If $E(\infty, M)$ converges, then $E(x, M)$ can be omitted from the error term. As a special case of this result, we show that

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