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ANOTHER FAMILY OF q-LAGRANGE INVERSION FORMULAS

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ABSTRACT. A q-analog of Lagrange inversion is stated for $(x/(1 - x^r)^b)$. Applications to basic hypergeometric series, identities of the Rogers-Ramanujan type, and orthogonal polynomials are given.

1. Introduction. The generalized Lagrange inversion problem is: given

$$G_k(x) = \sum_{n=k}^{\infty} B_{nk}x^n, \quad k = 0, 1, \ldots, \tag{1.1}$$

for some lower triangular non-singular matrix B_{nk}, and a formal power series

$$f(x) = \sum_{n=0}^{\infty} f_n x^n, \tag{1.2}$$

find constants a_k such that

$$f(x) = \sum_{k=0}^{\infty} a_k G_k(x). \tag{1.3}$$

It is clear that

$$f_n = \sum_{k=0}^{n} B_{nk}a_k. \tag{1.4}$$

Thus to find a_k it is sufficient to find the inverse matrix $B_{k\ell}^{-1}$:

$$a_k = \sum_{\ell=0}^{k} B_{k\ell}^{-1} f_\ell. \tag{1.5}$$

The usual Lagrange inversion formula takes $G_k(x) = y^k$, where $y(x)$ is a formal power series in x such that $y(0) = 0$ and $y'(0) \neq 0$.

In a recent paper **[10]** we gave a q-analog of B_{nk}, $B_{k\ell}^{-1}$, and $G_k(x)$ for $G_k(x) = x^k/(1 - x)^{a+(b+1)k}$. In this paper we similarly find a q-Lagrange inversion formula for a q-analog of $G_k(x) = x^k/(1 - x^r)^{a+(b+1)k}$ for $r = 1, 2, \cdots$. Our main theorem is stated as Theorem 2.3. Just as in **[10]**,

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