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ON FACTORIZATION OF BIINFINITE TOTALLY POSITIVE BLOCK TOEPLITZ MATRICES

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ABSTRACT. We extend the work of Aissen, Schoenberg, Whitney, and Edrei, who characterized the symbol of a totally positive Toeplitz matrix, to characterizing the determinant of the symbol of a block Toeplitz totally positive matrix. As a consequence of our arguments we show that the symbol for the block Toeplitz case may be factored just as in the Toeplitz setting.

1. Introduction. A biinfinite matrix $A = (A_{ij})$, $-\infty < i, j < \infty$, is called totally positive provided that all its minors are nonnegative, i.e., for all $i_1 < \cdots < i_p, j_1 < \cdots < j_p$

$$A\begin{pmatrix} i_1, \ldots, i_p \\ j_1, \ldots, j_p \end{pmatrix} = \begin{vmatrix} A_{i_1,j_1} & \cdots & A_{i_1,j_p} \\ \vdots & & \vdots \\ A_{i_p,j_1} & \cdots & A_{i_p,j_p} \end{vmatrix} \geqq 0. \tag{1.1}$$

In two previous papers, [**5**, **6**], we were concerned with factorization and invertibility of such matrices. Our motivation for these questions arose from certain problems in the theory of spline functions. Consequently, these papers only treated the case where A is banded, i.e., for some integers n and m, with m nonnegative, $A_{ij} = 0$, if $i - j < n$ or $i - j > n + m$. In this case, we say A is m-banded. We will concentrate our attention here on matrices which are block Toeplitz. Thus for some integer N we require

$$A_{ij} = A_{i+N,j+N} \text{ all } i, j \in \mathbf{Z}.$$

Any such matrix has the block structure

$$A = \begin{bmatrix} & & \cdot & \cdot & & \cdot & \\ \cdots & A_{-1} & A_0 & A_1 & A_2 & \cdot & \cdots \\ \cdots & A_{-2} & A_{-1} & A_0 & A_1 & A_2 & \cdots \\ \cdots & \cdot & A_{-2} & A_{-1} & A_0 & A_1 & \cdots \\ & & \cdot & \cdot & \cdot & & \end{bmatrix} \tag{1.2}$$

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