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THE ORDER OF MAGNITUDE OF THE (C, $\alpha \geqq 0$, $\beta \geqq 0$)-MEANS OF DOUBLE ORTHOGONAL SERIES

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1. Introduction. Let (X, F, μ) be an arbitrary positive measure space and $\{\phi_{ik}(x): i, k = 1, 2, \ldots\}$ an orthonormal system on X. We consider the double orthogonal series

$$(1.1) \qquad \sum_{i=0}^{\infty} \sum_{k=0}^{\infty} a_{ik}\phi_{ik}(x),$$

where $\{a_{ik}: i, k = 0, 1, \ldots\}$ is a sequence of real numbers for which

$$(1.2) \qquad \sum_{i=0}^{\infty} \sum_{k=0}^{\infty} a_{ik}^2 < \infty.$$

We denote by $s_{mn}(x)$ the rectangular partial sums of series (1.1):

$$s_{mn}(x) = \sum_{i=0}^{m} \sum_{k=0}^{n} a_{ik}\phi_{ik}(x) \qquad (m, n = 0, 1, \ldots).$$

Let α and β be real numbers, $\alpha > -1$ and $\beta > -1$. We recall that the (C, α, β)-means of series (1.1) are defined

$$\sigma_{mn}^{\alpha\beta}(x) = \frac{1}{A_m^\alpha A_n^\beta} \sum_{i=0}^{m} \sum_{k=0}^{n} A_{m-i}^{\alpha-1} A_{n-k}^{\beta-1} s_{ik}(x)$$
$$= \frac{1}{A_m^\alpha A_n^\beta} \sum_{i=0}^{m} \sum_{k=0}^{n} A_{m-i}^{\alpha} A_{n-k}^{\beta} a_{ik}\phi_{ik}(x), \qquad (m, n = 0, 1, \ldots),$$

where

$$A_m^\alpha = \binom{m+\alpha}{m} = \begin{cases} (\alpha+1)(\alpha+2)\cdots(\alpha+m)/m!, & \text{for } m = 1, 2, \ldots, \\ 1, & \text{for } m = 0, \end{cases}$$

(see, e.g., [**9**, p. 77]).

The case $\alpha = \beta = 0$ gives back the rectangular partial sums $s_{mn}(x) = \sigma_{mn}^{00}(x)$. The case $\alpha = \beta = 1$ gives the first arithmetic means with respect to m and n,

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