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SOME CHOQUET THEOREMS*

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ABSTRACT. A geometric and equivalent version of a fundamental analytic compact Choquet theorem is proved. This new theorem is then strengthened and, furthermore, an analogous non-compact version of this last geometric theorem is demonstrated. These theorems introduce a new point of view. Other results are proved as well.

1. Introduction. Fundamental to a good part of Choquet's representation theory are the Choquet-Bishop-de Leeuw and Choquet-Meyer theorems, since the former ensures the existence of representing boundary measures in the important geometric compact case, and the latter gives necessary and sufficient conditions for the uniqueness of such representation. More precisely, these theorems state that:

THEOREM 1.1. (CHOQUET-BISHOP-DE LEEUW). *Let K be a compact convex subset of a (real or complex) locally convex topological vector space. Then every point of K is the barycenter of a maximal probability measure on K.*

THEOREM 1.2. (CHOQUET-MEYER). *Every point of K is the barycenter of a unique maximal probability measure on $K \Leftrightarrow K$ is a simplex.*

If E denotes the locally convex topological vector space that contains K, and E^* its topological dual, then the point $x \in K$ is said to be the barycenter of μ (a probability measure on K) provided x is the resultant of μ (written $x = r(\mu)$); i.e., $\ell(x) = \int_K \ell|_K \, d\mu \ \forall \ell \in E^*$ [12]. Denote $\{\rho \in \mathbf{R}: \rho \geqq 0\}$ by $\mathbf{R}^+$; a convex subset C of a real or complex vector space V is said to be a simplex $\Leftrightarrow$ the proper cone $\mathbf{R}^+(C \times 1)$ is a lattice in the partial ordering which it induces on $V \times \mathbf{R}$. Probability and maximal measures are defined in the first of the next section definitions; almost all the notational conventions used in this paper are explained in the definitions of (and elsewhere in) the following section. More ample in-

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