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A GENERALIZATION OF A RESULT OF A. MEIR FOR NON-DECREASING SEQUENCES

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1. In [3], the following result is given.

THEOREM A. *Let* $0 \leqq p_1 \leqq p_2 \leqq \cdots \leqq p_n$ *and* $0 = a_0 \leqq a_1 \leqq \cdots \leqq a_n$, *satisfying* $a_i - a_{i-1} \leqq p_i$ $(i = 1, 2, \ldots, n)$. *If* $r \geqq 1$ *and* $s + 1 \geqq 2(r + 1)$, *then*

$$(1.1)\quad \left((s+1)\sum_{i=1}^{n-1} a_i^s \frac{p_i + p_{i+1}}{2}\right)^{1/(s+1)} \leqq \left((r+1)\sum_{i=1}^{n-1} a_i^r \frac{p_i + p_{i+1}}{2}\right)^{1/(r+1)}.$$

In this paper we shall prove an inequality which is stronger than inequality (1.1). Also, we show a generalization of Theorem A.

THEOREM 1. *Let* $0 \leqq p_1 \leqq p_2 \leqq \cdots \leqq p_n$ *and* $0 = a_0 \leqq a_1 \leqq \cdots \leqq a_n$, *satisfying* $a_i - a_{i-1} \leqq p_i (i = 1, 2, \ldots, n)$. *If* $r \geqq 1$ *and* $s + 1 \geqq 2(r + 1)$, *then*

$$(1.2)\quad \begin{aligned} &(s+1)\sum_{i=1}^{n-1} a_i^s \frac{p_i + p_{i+1}}{2} + \frac{(s+1)(s-r)}{8}\sum_{i=1}^{n-1}(p_{i+1}^2 - p_i^2)a_i^{s-1} \\ &\leqq \left((r+1)\sum_{i=1}^{n-1} a_i^r \frac{p_i + p_{i+1}}{2}\right)^{(s+1)/(r+1)}. \end{aligned}$$

2. PROOF. Since $x \mapsto x^r$ $(r \geqq 1)$ is a convex function on $[0, \infty)$, the inequality

$$\sum_{i=1}^{j}\int_{a_{i-1}}^{a_i} x^r dx \leqq \sum_{i=1}^{j}(a_i - a_{i-1})\frac{a_i^r + a_{i-1}^r}{2} \quad (1 \leqq j \leqq n),$$

holds, wherefrom, according to the condition $a_i - a_{i-1} \leqq p_i$, we obtain

$$(2.1)\quad \frac{1}{r+1}a_j^{r+1} \leqq \sum_{i=1}^{j} p_i \frac{a_i^r + a_{i-1}^r}{2}.$$

For $q_j = \sum_{i=1}^{j} a_{i0}^r \frac{p_i + p_{i+1}}{2}$, the inequality (2.1) becomes

$$\frac{1}{r+1}a_j^{r+1} \leqq q_j - \frac{1}{2}p_{j+1}a_j^r = q_{j-1} + \frac{1}{2}p_j a_j^r,$$

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