ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 16, Number 2, Spring 1986

ON BOUNDARY VALUES OF SOLUTIONS OF A QUASI-LINEAR PARTIAL DIFFERENTIAL EQUATION OF ELLIPTIC TYPE

J. CHABROWSKI

Introduction. In this article we study traces of generalized solutions of quasi-linear elliptic equations. We obtain a sufficient condition for a solution in $W^{1,2}_{\mathrm{loc}}(Q)$ to have an L^2-trace on the boundary. The results are then applied to establish an existence theorem for the Dirichlet problem. The arguments which we give here are based partially on the references **[2]** and **[4]**.

The outline of this paper is as follows. §1 contains preliminary work. §2 deals with the problem of traces for solutions in $W^{1,2}_{\mathrm{loc}}(Q)$. The main result here is Theorem 1, which justifies the approach to the Dirichlet problem adopted in §4. In §3 we derive an energy estimate for solutions of the Dirichlet problem with L^2-boundary data.

1. Preliminaries. Consider the quasi-linear elliptic equation of the form

$$(1) \qquad -\sum_{i,j=1}^{n} D_i(a_{ij}(x, u)D_j u) + b(x, u, Du) = 0$$

in a bounded domain $Q \subset R_n$ with the boundary ∂Q of the class C^2, $Du = (D_1 u, \ldots, D_n u)$, $D_i u = \partial u/\partial x_i$.

Throughout this paper we make the following assumptions.

(A) There is a positive constant γ such that

$$\gamma^{-1}|\xi|^2 \leqq \sum_{i,j=1}^{n} a_{ij}(x, u)\xi_i\xi_j \leqq \gamma|\xi|^2,$$

for all $\xi \in R_n$ and $(x, u) \in Q \times (-\infty, \infty)$; moreover, $a_{ij}(x, u)$ are uniformly continuous on $\bar{Q} \times (-\infty, \infty)$ and, for every $u \in (-\infty, \infty)$, $a_{ij}(\cdot, u) \in C^1(\bar{Q})$ $(i, j = 1, \ldots, n)$, and there exists a positive constant K such that $|D_i a_{ij}(x, u)| \leqq K$, for all $(x, u) \in Q \times (-\infty, \infty)$, $a_{ij} = a_{ji}$ $(i, j = 1, \ldots, n)$.

(B) The function $b(x, u, s)$ is defined for $(x, u, s) \in Q \times R_{n+1}$, $s = (s_1, \ldots, s_n)$, and satisfies the Carathéodory condition:

(i) for a.e. $x \in Q$, $b(x, \cdot, \cdot)$ is a continuous function on R_{n+1}; and

Received by the editors on July 27, 1984.
Copyright © 1986 Rocky Mountain Mathematics Consortium