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ON THE QUARTIC CHARACTER OF CERTAIN QUADRATIC UNITS AND THE REPRESENTATION OF PRIMES BY BINARY QUADRATIC FORMS

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1. For a squarefree rational integer $m > 1$ let ε_m be the fundamental unit of $\mathbf{Q}(\sqrt{m})$ normalized by $\varepsilon_m > 1$. For a rational prime $p \equiv 1 \bmod 4$ let $(\cdot/p)$ be the quadratic and $(\cdot/p)_4$ the quartic residue symbol modulo p. It is the aim of this paper to prove the following conjecture of P. A. Leonard and K. S. Williams ([**8**, Conjecture 3.6]):

THEOREM. *Let q, q' be primes, $q \equiv 3 \bmod 8$, $q' \equiv 7 \bmod 8$, $(q'/q) = 1$, and let s be the odd part of the class number of $\mathbf{Q}(\sqrt{qq'}, \sqrt{-2})$. Let p be a prime such that $(-1/p) = (2/p) = (q/p) = (q'/p) = 1$; then*

$$p^s = x^2 + 8qq'y^2 = c^2 + 8d^2$$

with $x, y, c, d \in \mathbf{Z}$ and

$$\left(\frac{\varepsilon_{qq'}}{p}\right)_4 = \left(\frac{\varepsilon_{2q'}}{p}\right)_4 \cdot (-1)^{y+d}.$$

REMARK 1. When proving the Theorem it will be shown that for the primes p in question $(\varepsilon_{qq'}/p) = (\varepsilon_{2q'}/p) = 1$ and that the quartic symbols are well defined.

REMARK 2. Perhaps the Theorem itself does not deserve an extra publication but the proof is an interesting journey through various branches of algebraic number theory and is intimately connected with the so-called explicit decomposition laws in algebraic number fields which are not yet fully understood.

2. The fields involved. I keep all notations of the Theorem and begin with the unit theory of the biquadratic field

$$K = \mathbf{Q}(\sqrt{qq'}, \sqrt{2q'}),$$

using methods and results of [7].

On account of $(q', qq'/p) = 1$ for all primes p, there is an integral $\delta_{qq'} \in \mathbf{Q}(\sqrt{qq'})$ with

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