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CLOSED MAPS AND SPACES WITH ZERO-DIMENSIONAL REMAINDERS

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ABSTRACT. A O-space is a completely regular Hausdorff space possessing a compactification with zero-dimensional remainder. It is well known that any O-space X possesses a maximum compactification F_0X having this property. The following question is considered: if f: $X \to Y$ is a closed map, and X, Y are O-spaces, under what conditions on X, Y and/or f will f extend to $g \in C(F_0X, F_0Y)$? It is proved that if Y is rimcompact, then it is (necessary and) sufficient that for any distinct pair of points $y, z \in Y$, $Cl_{F_0X}f^{\leftarrow}(y) \cap Cl_{F_0X}f^{\leftarrow}(z)=\emptyset$. This result is used to show that if i) X is a realcompact or metacompact O-space and Y is a rimcompact space in which the set of q-points has discrete complement, or if ii) X is a metacompact O-space or a locally compact realcompact space, and Y is a rimcompact k-space, then any closed map from X into Y extends to a map from F_0X into F_0Y.

1. Introduction and known results. A 0-space is a completely regular Hausdorff space possessing a compactification with zero-dimensional remainder. Such a compactification will be called zero-dimensional at infinity (denoted by O.I.). Any 0-space X possesses a maximum O.I. compactification ([11]) which we denote by F_0X. (A discussion of the standard partial ordering on the compactifications of X appears below.)

Various researchers have considered the following question. If X, Y are 0-spaces, and f: $X \to Y$ is a closed map, under what conditions on X, Y and/or f will f extend to $g \in C(F_0X, F_0Y)$? Recall that a space is rimcompact if it has a basis of open sets with compact boundaries ([9]). Any rimcompact space is a 0-space; the converse is not true ([17]). In Lemma 1 of [5] it is shown that if X is rimcompact, $f \in C(X, [0, 1])$, and the set $\{y \in [0, 1]: f^{\leftarrow}(y)$ contains a compact set K such that $X \backslash K$ can be written as $U \cup V$, where U, V are π-open in X and $U \subset f^{\leftarrow}[0, y]$, while $V \subset f^{\leftarrow}[y, 1]\}$ is dense in [0, 1], then f extends to $g \in C[F_0X, [0, 1])$. An argument in the proof of Theorem 3 of [15] shows that if f: $X \to Y$ is closed, X and Y are rimcompact and $\mathrm{bd}_X f^{\leftarrow}(y)$ is compact for each $y \in Y$,

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