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A Q-ANALOGUE OF APPELL'S F_1 FUNCTION AND SOME QUADRATIC TRANSFORMATION FORMULAS FOR NON-TERMINATING BASIC HYPERGEOMETRIC SERIES

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ABSTRACT. A q-analogue of the integral representation of Appell's F_1 function is given as an extension of Askey and Wilson's q-beta integral and is evaluated as a sum of three very well-poised ${}_{10}\phi_9$ series. The formula is then applied to find two different types of quadratic transformation formulas between very well-poised ${}_{10}\phi_9$ series and balanced ${}_5\phi_4$ series. Special cases of balanced and very well-poised ${}_{10}\phi_9$ series are also examined.

1. Introduction. The Appell function F_1 is defined by the double infinite series [7, p. 224]

$$(1.1)\qquad F_1(\alpha, \beta, \beta', \gamma; x, y) = \sum_m \sum_n \frac{(\alpha)_{m+n}(\beta)_m(\beta')_n}{m!n!(\gamma)_{m+n}} x^m y^n$$

subject to usual convergence restrictions, where the shifted factorials are defined by $(a)_0 = 1$, $(a)_m = a(a+1)\cdots(a+m-1)$, $m = 1,2,\ldots$. Of all the Appell functions this is the only one that has a representation in terms of a single integral [7, p. 231]

$$(1.2)\qquad \begin{aligned} &F_1(\alpha, \beta, \beta', \gamma; x, y) \\ &= \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma-\alpha)} \int_0^1 t^{\alpha-1}(1-t)^{\gamma-\alpha-1}(1-xt)^{-\beta}(1-yt)^{-\beta'}dt, \end{aligned}$$

where $0 < \operatorname{Re}\alpha < \operatorname{Re}\gamma$. Using the q-beta type integral of Askey and Wilson [3] the authors [8] recently found the following q-analogue of Euler's integral representation of Gauss' hypergeometric series ${}_2F_1$

$$(1.3)\qquad \begin{aligned} &{}_8\phi_7\left[\begin{matrix}\lambda abcq^{-1}, q\sqrt{\ }, -q\sqrt{\ }, bc, ac, ab, \lambda d^{-1}, \lambda f^{-1} \\ \sqrt{\ }, -\sqrt{\ }, \lambda a, \lambda b, \lambda c, abcd, abcf\end{matrix}; q, df\right] \\ &= \frac{(q, ab, ac, ad, af, bc, bd, bf, cd, cf, \lambda abc; q)_\infty}{2\pi(\lambda a, \lambda b, \lambda c, abcd, abcf; q)_\infty} \\ &\quad\cdot \int_{-1}^1 w(z; a, b, c, d) \frac{h(z; \lambda)}{h(z; f)} dz, \end{aligned}$$

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