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WHY SECOND ORDER PARABOLIC SYSTEMS?

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0. Introduction. A natural phenomenon is envisaged, describable by a set of functions

$$\rho_i(x, t), \qquad 1 \leqq i \leqq m,$$

subject to some evolutionary law. Here, x is interpreted as a space variable and the $\rho_i(x, t)$ as the concentration of "species" $i = 1, \ldots, m$ at x and at time t. We ask for certain "first experiments" which permit us to conclude that the evolutionary law governing the envisaged phenomenon is a system of partial differential equations of parabolic type independent of the initial distribution $\rho(x, 0)$. These "first experiments" do not necessarily have to be real experiments, but may be any source of information.

We shall, in fact, provide a set of general properties listed below as $A_1, A_2, \ldots$, which in a purely mathematical way imply that the $\rho_i(x, t)$ solve a system of equations of the form

$$(*) \quad \frac{\partial}{\partial t}\rho_j(x, t) = \sum_{i,k,\ell} a^{i,k}_{j,\ell}(x)\frac{\partial^2}{\partial x_i \partial x_k}\rho_\ell(x, t) + \sum_{i,\ell} b^i_{j,\ell}(x)\frac{\partial}{\partial x_i}\rho_\ell(x, t) + F_j(x, \rho(x, t)) \; i, k \in \{1, 2, \ldots, n\}, j, \ell \in \{1, 2, \ldots, m\}.$$

Some of the properties $A_1, A_2, \ldots$ are in fact necessary for a process to satisfy such a system of equations. A particularly simple property considered is

A_6: If $\rho_i(x, 0)$ is nonnegative for all x and i, then $\rho_i(x, t)$ is nonnegative for all x, i and $t \geqq 0$.

In compiling our set of assumptions $A_1, A_2, \ldots$, we have tried to make them as simple and as few in number as possible, as well as being subject to actual verification by measurement.

Once the form of the evolutionary law governing the envisaged process is determined to be (*), one can try to find the coefficients

$$a^{i,k}_{j,\ell}, \; b^i_{j,\ell} \cdots$$

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