

AN ELEMENTARY APPROACH TO THE MULTIPLICITY THEORY OF MULTIPLICATION OPERATORS

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1. Introduction. The spectral theory of normal operators comprises one of the prettier and more complete chapters in the literature of operators on Hilbert space. The high point of this development is the multiplicity theory, due in original form to Hellinger and subsequently cast in the language of Lebesgue integration by Hahn [18], which answers for normal operators one of the favorite questions of any mathematician: how do you tell when two of these objects are “the same”, that is, equivalent in the appropriate sense.

Present day multiplicity theory comes in several levels of abstraction, generality and sophistication through the efforts of a number of mathematicians; see [7, 10, 11, 12, 16] and the further references there. The most straightforward approach to the separable theory can easily make an appearance (and often does) in a first graduate course in functional analysis and operator theory, say just after the spectral theorem. The kind of treatment I have in mind can be found in an eminently readable book by Conway [9]. Of course, every good theory deserves some meaty but accessible examples, and there’s the rub, pedagogically speaking: the multiplicity theory of concrete normal operators is often either trivial or quite hard to compute. The class of examples that surely comes first to mind, multiplication operators on L^2 spaces, was worked out only relatively recently [2]; the treatment given there uses in a crucial way one first of the more highbrow versions of multiplicity theory (direct integrals, replete with measurable fields of Hilbert spaces) plus a theorem on disintegration of measures. It is the purpose of this note to present a straightforward, rather bare-hands calculation of the spectral multiplicity function of a multiplication operator on $L^2(0, 1)$ which is compatible with the more elementary general theory and which, I hope, will be accessible to students seeing these ideas for the first time. The train of thought here has also been used by the author [15] and Ball [5] to analyze more complicated operators.

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