ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 16, Number 1, Winter 1986

THE CHARACTERIZATION OF DEGENERATE AND NON-DEGENERATE SYSTEMS

ALAN R. HAUSRATH AND RAUL F. MANASEVICH

I. Introduction. In this paper we study the system

$$u' = F(u), \tag{1.1}$$

where $F:U \to \mathbf{R}^2$, $U \subset \mathbf{R}^2$ is open, $0 \in U$ and $F(0) = 0$. We assume that F is C^1 and that the origin is a center of (1.1).

Let $v(t)$ be a non-constant T-periodic solution of (1.1) and consider the corresponding linear variational equation

$$y' = F_u(v(t))y. \tag{1.2}$$

DEFINITION 1.1. We say that v is degenerate if and only if every solution of the corresponding linear variational equation (1.2) is T-periodic.

Since $y = v(t)$ is a T-periodic solution of (1.2) we have that v will be degenerate if and only if there exists a T-periodic solution of (1.2) that is linearly independent of $v(t)$.

DEFINITION 1.2. We say that (1.1) is degenerate in a neighborhood of 0, or simply degenerate, if and only if every non-constant periodic solution in this neighborhood is degenerate.

DISCUSSION. We will see that (1.1) is non-degenerate if and only if the periodic solutions in a neighborhood of 0 have distinct minimum periods, for example, as a function of the maximum amplitude of the solution. Thus, this concept is a generalization of the idea of "hard" and "soft" springs for the equation of a nonlinear spring $x'' + g(x) = 0$. Although this idea is interesting in its own right, it also has important applications in the study of

$$x' = F(x) + \varepsilon g(t, x), \tag{1.3}$$

where g is T-periodic in t. For example, if (1.1) is Hamiltonian and non-

Received by the editors on March 5, 1984 and in revised form on October 3, 1984.
Sponsored in part by B.S.U. Research Grant 681A015 for the first author and by DIB, University of Chile, Research Grant E-14268433 and CONICYT for the second author.