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ON FACTORIZATION OF OPERATOR POLYNOMIALS AND ANALYTIC OPERATOR FUNCTIONS

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ABSTRACT It is proved that the set of biquasitriangular monic operator polynomials which admit factorization into monic linear factors, is dense in the set of all biquasitriangular monic operator polynomials. An extension of this result to the factorization of analytic operator functions with compact spectrum is obtained as well. These results generalize a known factorization property of monic matrix polynomials.

1. Introduction. Let $L(\lambda) = \sum_{j=0}^{\ell} \lambda^j A_j$ be a ploynomial whose coefficients A_j are (linear bounded) operators $H \to H$, where H is a fixed separable (complex) Hilbert space. We shall assume always that the operator polynomial $L(\lambda)$ is monic, i.e., with leading coefficient $A_\ell = I$. The problem of factorization of $L(\lambda)$ into a product of several operater polynomials is an important one and has attracted much attention recently. This problem was studied in [**9**] in connection with oscillations of continua, and in [**12**, **1**, **4**, **11**] (the list is far from being complete). In case H is finite dimensional, a comprehensive treatment of this problem can be found in [**3**].

It turns out that, in case $H = \mathbf{C}^n$, not every monic operator polynomial $L(\lambda) = \sum_{j=0}^{\ell} \lambda^j A_j$, admits a factorization into a product of linear factors

$$L(\lambda) = (\lambda I + X_1)(\lambda I + X_2) \cdots (\lambda I + X_\ell), \tag{1}$$

where $X_\ell : \mathbf{C}^n \to \mathbf{C}^n$ are operators (unless, of course, $n = 1$). However, if the companion operator

$$C_L = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & & & \vdots \\ 0 & 0 & & \cdots & I \\ -A_0 & -A_1 & & \cdots & -A_{\ell-1} \end{bmatrix} : \mathbf{C}^{n\ell} \to \mathbf{C}^{n\ell}$$

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