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THE SPACE $C_0(p)$ OVER VALUED FIELDS

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Introduction. Let K be a non-trivially (rank 1) valued complete field (if necessary we shall specify the nature of the valuation depending on the context). For a sequence $p = \{p_k\}$ of positive real numbers, we define the space

$$C_0(p) = \{x = \{x_k\}: x_k \in K, |x_k|^{p_k} \to 0, k \to \infty\},$$

where $|\cdot|$ denotes the valuation on K. Clearly, $C_0(p)$ is a linear space if and only if p is a bounded sequence and we shall assume henceforth that p is a bounded sequence without explicit mention. Define

$$g(x) = \sup_{k\geq 1} \{|x_k|^{p_k/H}\}, \qquad H = \max(1, \sup_{k\geq 1} p_k).$$

Then g defines a paranorm on $C_0(p)$ and so $d(x, y) = g(x - y)$ defines a metric on $C_0(p)$ with respect to which $C_0(p)$ is a complete metric linear space. On the other hand, we can also define seminorms

$$\mathscr{P}_n(x) = \sup_{k\geq 1} \{|x_k|n^{1/p_k}\}, \qquad n = 1, 2, \ldots, x \in C_0(p),$$

so that the metric d is compatible with the locally convex (locally K-convex) topology defined by these seminorms. In other words, $C_0(p)$ is a Frechet space. Furthermore, the dual $C_0(p)^*$ of $C_0(p)$ consists of functionals f given by:

i) $f(x) = \sum_{k=1}^{\infty} a_k x_k$, $a_k \in K$ such that $\sum_{k=1}^{\infty} |a_k|\, N^{-1/p_k} < \infty$ for some $N > 1$, when the valuation is archimedean (see [6]);

ii) $f(x) = \sum_{k=1}^{\infty} a_k x_k$, $a_k \in K$ such that $\sup_{k\geq 1} |a_k|^{p_k} < \infty$ when the valuation is non-archimedean.

Also, $C_0(p)^*$ is endowed with the topology of uniform convergence over bounded subsets of $C_0(p)$. We write

$$1_\infty(p) = \{\{x_k\}, x_k \in K, \sup_{k\geq 1} |x_k|^{p_k} < \infty\}.$$

(*) The results obtained by the second author independently of the first, and now presented for publication jointly, were included in his thesis approved for Ph. D. by the University of Madras in March 1981.

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