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STATIONARY SPACIAL PATTERNS FOR A REACTION-DIFFUSION SYSTEM WITH AN EXCITABLE STEADY STATE

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ABSTRACT. In this note, the existence of stationary patterns in $n \geqq 2$ dimensional state space is established for a reaction-diffusion system which exhibits a single-globally attracting, excitable steady state. The system studied is dynamically like the FitzHugh-Nagumo model for nerve conduction but has a large inhibitor diffusion term. Variational methods are applied to an energy functional which give one pattern as a minimum and a second as a saddle point of the functional.

1. Introduction. Consider the system

$$
(1.1) \qquad \begin{aligned} u_t &= \Delta u + f(u) - v \\ v_t &= D\,\Delta v + \varepsilon(u - \gamma v) \end{aligned},
$$

where $\Delta \equiv \sum_{i=1}^{n} \partial^2/(\partial x_i^2)$, $n \geqq 1$, $t \geqq 0$, $(x_1, \ldots, x_n) \in \Omega \subseteq \mathbf{R}^n$, $f(u) = u(1-u)(u-a)$, $0 < a < 1/2$, $D > 0$, $\varepsilon > 0$, and $\gamma > 0$. Equations (1.1) are an extension of the simpler FitzHugh-Nagumo [**3**, **10**] equations, namely

$$
(1.2) \qquad \begin{aligned} u_t &= u_{xx} + f(u) - v \\ v_t &= \varepsilon(u - \gamma v). \end{aligned}
$$

The FitzHugh-Nagumo system serves as a prototype for nerve conduction and other chemical and biological systems. The interested reader is referred to [**6**, **11**] for a review of results obtained to this date.

Recently, Ermentrout, Hastings and Troy [**2**] have proposed system (1.1) as a prototype model for systems which exhibit lateral inhibition and excitability. In this setting u is interpreted as an activator concentration and v is interpreted as an inhibitor concentration. They discuss the physical motivation for the existence of nonconstant stable time independent solutions of (1.1) when $n = 1$ and solutions u and v are defined on all of $\mathbf{R}$ with $u(\pm\infty) = v(\pm\infty) = 0$. Summarizing their discussion, if $0 < \gamma < 4/(1-a)^2$ then the dynamic equations

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