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ON REGULAR GROWTH AND ASYMPOTIC STABILITY

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We are concerned with the global asymptotic stability of the trivial solution to the question

$$(1) \qquad y'' + q(x)y = 0, \qquad 0 \leqq x \leqq X,$$

when $q(x)$ is right-continuous, nondecreasing, and $\lim_{x \to X} q(x) = +\infty$. It is well known that (1) possesses a zero-tending solution under these assumptions, and examples exist which show that not all solutions need satisfy

$$(2) \qquad \lim_{x \to X} y(x) = 0.$$

What is needed is an extra assumption preventing $q(x)$ from doing most of its growth on arbitrarily small sets, i.e., a regular growth assumption. The paper of Macki **[1]** surveys the various distinct notions of regular growth. In this paper we present a definition of regular growth which improves and unifies the various existing concepts.

Such regular growth conditions are obtained by a converse path, by assuming that (1) has a solution not satisfying (2), and making deductions about the behaviour of $q(x)$ in relation to certain sequences of x-values. These deductions, when used as hypotheses, then form necessary conditions for (1) to have a solution not satisfying (2) and provide a concept of irregular growth. The more such deductions are utilised in this way, the narrower may be the class of $q(x)$ of irregular growth, and so the wider the class of $q(x)$ of regular growth for which (2) must be the case.

We shall work in terms of various notions of "irregular growth". Roughly speaking, these have the character that there must be a family of sequences satisfying certain conditions (see, e.g., (i), (iii), (iv), (v) below) for which a certain series (see (ii)) converges. The converse notion of regular growth, as generally presented in the literature, takes the form that, for every such set of sequences, the series in question must diverge. We will not present these converse notions explicitly here, since they are obtainable by immediate translation from the associated con-

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