ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 16, Number 1, Winter 1986

PARALLEL MAPS THAT PRESERVE GEOMETRIC OBJECTS OF HYPERSURFACES

KRISHNA AMUR

ABSTRACT. It is known that parallel maps of hypersurfaces in R^{n+1} preserve principal directions, umbilics and the third fundamental form [4]. We study the conditions under which the parallel map $f_t^\#$ of a parallel Σ_{-t} of a hypersurface Σ into the parallel Σ_t preserves other geometric objects besides the three mentioned above and show, in particular, that when the determinant of the Jacobian matrix of $f_t^\#$ is 1 and n is even, Σ is a certain non-trivial minimal hypersurface and $f_t^\#$ preserves the element of area and all the even order elementary symmetric functions of principal curvatures.

Introduction. Let Σ_t and Σ_{-t} denote parallel hypersurfaces of an immersed hypersurface Σ in R^{n+1} for a sufficiently small parameter t. The parallel maps of Σ into Σ_{-t} and Σ_t, which we can assume to be local diffeomorphisms, define a parallel map $f_t^\#$ of Σ_{-t} into Σ_t. As a parallel map $f_t^\#$ preserves principal directions, umbilics, and the third fundamental form. In this paper we investigate the conditions under which other geometric objects of the hypersurfaces besides the three mentioned above are preserved by $f_t^\#$ and show that they occur in the form of restrictions on the non-singular Jacobian matrix of $f_t^\#$. We illustrate the use of such conditions in the proof of our main results stated in Proposition 2.1.

1. Parallel immersions. Let M be a connected, orientable smooth manifold of dimension n. Let $X: M \to R^{n+1}$ be an immersion. For sufficiently small values of t, the mappings $X_t, X_{-t}: M \to R^{n+1}$, defined by

$$(1.1)\quad X_t(p) = X(p) + t\,N(X(p)), \qquad X_{-t}(p) = X(p) - t\,N(X(p)),$$

where $p \in M$ and N is a unit normal vector field on $X(M)$, are also imimersions. Let $X(M) = \Sigma$, $X_t(M) = \Sigma^t$ and $X_{-t}(M) = \Sigma_{-t}$. Define f_t: $\Sigma \to \Sigma_t$ and f_{-t}: $\Sigma \to \Sigma_{-t}$ by

$$(1.2)\qquad f_t \circ X(p) = X_t(p), \qquad f_{-t} \circ X(p) = X_{-t}(p),$$

for all $p \in M$. We assume f_t and f_{-t} are local diffeomorphisms.

Received by the editors on January 23, 1984 and in revised form on May 30, 1984.
The work was supported by U. S. Government grant CIES 04946 under the Indo American Fellowship program and was carried out at North Carolina University at Chapel Hill.