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TRIPLE PRODUCTS IN THE GATEGORY OF SPECTRA OVER A SPACE

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ABSTRACT. We construct the category of spectra over a space and define triple products in this category. We prove that the semitensor product $H^*(B;Z_p) \circledcirc \mathscr{A}(p)$ may be interpreted as a certain set of morphisms in this category; as a consequence we define and discuss triple products in $H^*(B;Z_p) \circledcirc \mathscr{A}(p)$.

1. Introduction. In this paper we shall study triple products in the category of spectra over a space B. This category is a joint generalization of the stable category of J.F. Adams [2] and the category of spaces over a space introduced intependently by J. James [5], J. McClendon [8], and others. The definition of triple products in this category is a straightforward generalization of our earlier definition of triple products in the ordinary stable category [4], which in turn was a generalization of some work of E. Spanier [10].

We shall also prove a theorem that interprets the semitensor product $H^*(B; Z_p) \odot \mathscr{A}(p)$ [6, 7] as a set of morphisms in the category of spectra over B. This enables us to define triple products in $H^*(B; Z_p) \odot \mathscr{A}(p)$, and to show that if X is a stable two-stage Postnikov system over B, then the action of $H^*(B; Z_p) \odot \mathscr{A}(p)$ on $H^*_B(X; Z_p)$ is related to the triple-product structure of $H^*(B; Z_p) \odot \mathscr{A}(p)$. We shall also give a relationship between triple products in $H^*(B; Z_p) \odot \mathscr{A}(p)$ and those in $\mathscr{A}(p)$, as defined in [4].

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2. The category of spectra over B. In this section we shall construct the category of spectra over a space B, and we shall define triple products in this category.

Let B be a space. Then a space over B is a space X together with a map $p:X \to B$. A pointed space over B is a space X together with two maps, $p:X \to B$ and $s:B \to X$, such that $ps = 1_B$. Let (X, p, s) and (Y, q, r) be

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