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INTERPOLATING SEQUENCES ON CURVES

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ABSTRACT. We establish a condition on boundary curves (ending at points) lying either in the unit disc or the upper half plane which implies that any consecutively separated sequence, in the hyperbolic distance, lying on one of these curves is an interpolating sequence for bounded holomorphic functions.

1. Introduction. In a previous paper by the authors and Charles Belna [1] certain geometric properties of a sequence $\{z_n\}$ were shown to be sufficient that $\{z_n\}$ be an interpolating sequence for the algebra of bounded holomorphic functions in the unit disc Δ or the upper half plane H in the complex plane. In this paper we are concerned with identifying a class of curves in either H or Δ such that any sequence on such a curve satisfying a minimal hyperbolic separation is an interpolating sequence. A sequence $\{z_n\}$ in Δ is an interpolating sequence for the algebra $H^\infty(\Delta)$ if, for each bounded sequence $\{w_n\}$, there exists a function $f \in H^\infty(\Delta)$ such that $f(z_n) = w_n$, for all n. For $\{z_n\}$ to be interpolating for $H^\infty(\Delta)$ it is necessary and sufficient that it be uniformly separated. That is

$$\inf_n \prod_{\substack{k=1 \\ k\neq n}}^{\infty} \chi(z_n, z_k) > 0,$$

where $\chi(z, w)$ is the pseudo-hyperbolic distance in Δ. In case the domain is the upper half plane H we replace Δ by H in the above, retaining $\chi(z, w)$ as notation for the pseudo-hyperbolic distance in H. The necessity was established independently by L. Carleson [2], W.K. Hayman [4] and D.J. Newman [6]; the sufficiency was proved by Carleson [2]. A sequence $\{z_n\}$ in Δ or H is called separated if

$$\inf_{n\neq m} \chi(z_n, z_m) > 0. \tag{1.0}$$

Recently Gerber and Weiss [3] introduced a class $\mathscr{C}$ of subsets of Δ defined by the property that a sequence lying in $A \in \mathscr{C}$ is interpolating if and only if it is separated. They gave various characterizations of the class $\mathscr{C}$, one of which is that, for any set $A \in \mathscr{C}$, the closure of A in the maximal ideal space M contains only nontrivial homomorphisms. Other characteriza-