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ON THE DIOPHANTINE EQUATION $1 + p^a = 2^b + 2^c p^d$

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ABSTRACT. In this paper the exponential Diophantine equation $1 + p^a = 2^b + 2^c p^d$, where a, b, c, d are non-negative integers and p is an odd prime, is studied. All solutions to the equation are found for which $p \leqq 499$. This work extends earlier work of the authors and J. L. Brenner.

1. Introduction. In this paper we consider the equation

$$1 + p^a = 2^b + 2^c p^d \tag{1}$$

where p is an odd prime and a, b, c and d are non-negative integers. This equation is of the form

$$1 + x = y + z, \tag{2}$$

or, more generally,

$$\sum X_i = 0, \tag{3}$$

where the primes dividing xyz in (2) and $\prod X_i$ in (3) are specified.

There has been very little work done in general to solve such Diophantine equations. For example the equation

$$1 + 2^a 3^b = 5^c + 2^d 3^e 5^f \tag{4}$$

is unsolved. Some of these equations have an infinite number of trivial solutions. (For example the equation (4) above has infinitely many solutions of the form $c = f = 0$, $a = d$, and $b = e$.) It is unknown whether such equations always have only a finite number of non-trivial solutions.

It follows from work of Dubois and Rhin [6] and Schlickewei [7] that the related equation $p^a \pm q^b \pm r^c \pm s^d = 0$ has only finitely many solutions when p, q, r and s are distinct primes. However, their methods do not seem to apply when the terms in the equation are not powers of distinct primes.

The authors and J. L. Brenner [1], [2], [4], [5] have recently developed techniques which solve such equations in some cases. These techniques

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