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ORTHOGONAL POLYNOMIALS AND MEASURES WITH END POINT MASSES

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ABSTRACT. Let a sequence of orthogonal polynomials with respect to a given measure $d\psi(x)$ be explicitly known and let a new measure $d\psi^*(x)$ be constructed from $d\psi(x)$ by adjoining a positive mass at one point. When $d\psi(x)$ corresponds to one of the classical orthogonal polynomials of Jacobi, Hermite or Laguerre, the orthogonal polynomials relative to $d\psi^*(x)$ have been found by H. L. Krall and others. Here we consider general $d\psi(x)$ and obtain formulas for constructing the polynomials associated with $d\psi^*(x)$. A number of nonclassical examples are explicitly given.

1. Introduction. There has been a renewal of interest in the question of orthogonal polynomial solutions to linear differential equations. Much of the recent work can be considered a continuation of the work of Bochner [3] and his characterization of the classical orthogonal polynomials as the only orthogonal polynomials which are eigensolutions of $Ly = \lambda_n y$, where L is a second order linear differential operator with polynomial coefficients independent of n. H. L. Krall [11], [12] extended Bochner's work to the fourth order case and found three new sequences of orthogonal polynomials. The spectral measures for two of these could be obtained from the measures for the Laguerre and certain special Jacobi polynomials, respectively, by adjoining mass at one end of the spectral intervals. The third measure was obtained from the Legendre measure by adjoining equal masses at each end of the spectral interval. The three sets of polynomials have been studied in some detail by A. M. Krall [10].

Littlejohn [13], [14], [15] has continued this study of $Ly = \lambda_n y$. He has found a 6th order case which has orthogonal polynomial solutions. These polynomials are orthogonal with respect to the measure obtained from the Legendre measure by adjoining unequal masses at each end of the spectral interval. Koornwinder [9] generalized these results by obtaining explicitly the polynomials which are orthogonal with respect to the measure obtained from the most general Jacobi measure by adjoining arbitrary masses at both ends of the spectral interval. Koornwinder also indicates that these polynomials satisfy an 8th order case of $Ly = \lambda_n y$. Hendriksen

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