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THE COEFFICIENTS OF THE INVERSE OF AN ODD CONVEX FUNCTION

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1. Background information. $\mathscr{P}$ is the class of functions regular and with positive real part in the open unit disk $\mathcal{U}$, $\mathcal{U} = \{z \in \mathbf{C}: |z| < 1\}$, having a series representation

$$P(z) = 1 + c_1 z + c_2 z^2 + \ldots, \qquad z \in \mathcal{U}. \tag{1.2}$$

The family $\mathscr{K}$ of regular convex functions of the form

$$f(z) = z + a_2 z^2 + a_3 z^3 + \cdots \tag{1.3}$$

is defined by the condition

$$\frac{zf''(z)}{f'(z)} + 1 \in \mathscr{P} \tag{1.4}$$

(see [**4**], for example).

In recent years the peculiar behavior of the coefficients of inverses of functions in $\mathscr{K}$ and in similar classes has attracted attention [**1**, **2**, **7**, **8**, **10**, **11**]. If the inverse of $f(z)$ in $\mathscr{K}$ is

$$\check{f}(w) = w + A_2 w^2 + A_3 w^3 \cdots, \tag{1.5}$$

then it has been shown ([**1**, **10**]) that $|A_k| \leqq 1$, $k = 2, 3, \ldots, 8$, but that there are members of $\mathscr{K}$ for which $|A_{10}| > 1$, [**7**]. The exact bound for $|A_9|$ appears to be unknown at this time.

The purpose of the present work is to examine the coefficients of (1.5) when $f(z)$ is an odd function in $\mathscr{K}$. Suppose then that

$$f(z) = z + b_3 z^3 + b_5 z^5 + \cdots \tag{1.6}$$

is an odd member of $\mathscr{K}$. Then its inverse

$$\check{f}(w) = w + B_3 w^3 + B_5 w^5 + \cdots \tag{1.7}$$

is likewise odd. In this case we may write (1.4) as

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