ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 15, Number 3, Summer 1985

# TOMOGRAPHIC RECONSTRUCTION OF FUNCTIONS FROM THEIR SINGULARITIES

JAMES V. PETERS

The general problem of reconstructive tomography is to determine a density function $f(x)$, defined on $\mathbf{R}^n$, from its integrals on lower dimensional manifolds of some fixed dimension. Integrating $f$ over hyperplanes has received considerable attention due to its relation to $x$-ray scanning in $\mathbf{R}^2$ and nuclear magnetic resonance scanning in $\mathbf{R}^3$ [**4**, **6**]. This leads to the consideration of the Radon transform of $f$, defined on $S^{n-1} \times \mathbf{R}^1$, by

$$\hat{f}(\theta, p) = \int_{\mathbf{R}^n} f(x)\,\delta(p - \langle\theta, x\rangle)dx. \tag{1}$$

Here, $\delta$ denotes the Dirac delta mass and $\langle\cdot, \cdot\rangle$is the usual inner product. Thus, $\hat{f}$ is the integral of $f$ over the hyperplane $\{x \mid \langle\theta, x\rangle = p\}$. It will be assumed throughout that the density function $f(x)$ is absolutely summable and compactly supported. This implies that, for each $\theta \in S^{n-1}$, $\hat{f}$ is defined for almost every $p$. Further, $\hat{f}$ is compactly supported on $S^{n-1} \times \mathbf{R}^1$.

Related to $\hat{f}$ is the so-called back projection $\check{f}$. This is defined on $\mathbf{R}^n$ by

$$\check{f}(x) = \int_{S^{n-1}} \hat{f}(\theta, \langle\theta, x\rangle d\theta. \tag{2}$$

The function $\check{f}$ is directly related to $f$ by the singular integral

$$\check{f}(x) = \Omega_{n-1} \int_{\mathbf{R}^n} \frac{f(y)}{\|x - y\|}\, dy, \tag{3}$$

where $\Omega_n$ denotes the surface area of the unit sphere in $\mathbf{R}^n$. The value of $\Omega_n$ may be computed from the identity $\Omega_n = 2\pi^{n/2}/\Gamma(n/2)$, where $\Gamma$ denotes the gamma function. It follows directly from (3) that $\check{f}$ is continuous whenever $f$ is. By (1), the same implication also holds between $\hat{f}$ and $f$. Since identical results hold for the order of differentiability, it is quite often assumed in the literature that $f$ is sufficiently smooth to justify inversion formulas such as those given in [**5**].

The point of view adopted in this paper is that physical densities have

Received by the editors on February 14, 1983 and in revised form on December 12, 1983.