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SOME ISOMORPHISM INVARIANTS OF INTEGRAL GROUP RINGS

STEVE KMET AND SUDARSHAN SEHGAL

Dedicated to the memory of E.G. Straus and R.A. Smith

1. Introduction. Let ZG be the ingegral group ring of a group G. Denote by $\{\gamma_i(G)\}$, and $\{\delta_i(G)\}$ the lower central series, and the derived series of G, respectively. Let us denote by $D_i(G)$ the ith dimension subgroup

$$D_i(G) = G \cap (1 + \Delta^i(G)),$$

where $\Delta(G)$ is the augmentation ideal of ZG. Suppose that the torsion elements of G form a subgroup $T = T(G)$. Then we write $T_1 = T$ and for $i \geqq 1$ we write

$$T_{i+1} = T_{i+1}(G) = [G, T_i(G)],$$

the group generated by all commutators $(g, t) = g^{-1}t^{-1}gt$, $g \in G$, $t \in T_i$. Our main result is

THEOREM A. *Suppose that G and H are groups such that the torsion elements T(G) and T(H) of G and H respectively form subgroups. Suppose* $ZG \simeq ZH$. *Then we have*

(1) $T_i(G)/T_{i+j}(G) \simeq T_i(H)/T_{i+j}(H)$ for $1 \leqq j \leqq i + 2$,

(2) $D_i(G) \cap T(G)/D_{i+j}(G) \cap T(G) \simeq D_i(H) \cap T(H)/D_{i+i}(H) \cap T(H)$ for $1 \leqq j \leqq i + 2$,

(3) $\gamma_i(T(G))/\gamma_{i+j}(T(G)) \simeq \gamma_i(T(H))/\gamma_{i+j}(T(H))$ for $1 \leqq j \leqq i$,

(4) $\delta_i(T(G))/\delta_{i+1}(T(G)) \simeq \delta_i(T(H))/\delta_{i+1}(T(H))$ for all i,

(5) $\delta_i(T(G))/[G, \delta_i(T(G))]' \simeq \delta_i(T(H))/[G, \delta_i(T(H))]'$ for all i.

As a special case we have the following result.

THEOREM B. *Suppose that G and H are torsion groups such that* $ZG \simeq ZH$. *Then we have*

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