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CHAPTER 12 OF RAMANUJAN'S SECOND NOTEBOOK: CONTINUED FRACTIONS

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> We take up something—we know it is finite; but as soon as we begin to analyze it, it leads us beyond our reason, and we never find an end to all its qualities, its possibilities, its powers, its relations. It has become infinite.
>
> Vivekananda

Dedicated to the memory of R.A. Smith and E.G. Straus

1. Introduction. In assessing the content of Ramanujan's first letter, dated January 16, 1913, to him, Hardy [**34**, p. 9] remarked ". . . but (1.10) – (1.12) defeated me completely; I had never seen anything in the least like them before. A single look at them is enough to show that they could only be written down by a mathematician of the highest class. They must be true because, if they were not true, no one would have had the imagination to invent them." These comments were directed at three continued fraction representations. Indeed, Ramanujan's contributions to the continued fraction expansions of analytic functions are one of his most spectacular achievements. The three formulas which challenged Hardy's acumen are not found in Chapter 12, but this chapter, which is almost entirely devoted to the study of continued fractions, contains many other beautiful and penetrating formulas. Unfortunately, Ramanujan left us no clues as to how he discovered these elegant continued fraction formulas. Especially enigmatic are the several representations for products and quotients of gamma functions. Three of the principal formulas involving gamma functions are Entries 34, 39, and 40. Entries 20 and 22, giving Gauss's and Euler's continued fractions, respectively, for a quotient of two hypergeometric functions, also play prominent roles. Several other formulas are dependent upon these five entries, and it may be helpful to schematically indicate these connections among entries.

The purpose of this paper is to prove each of the 113 theorems, corol-

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