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ON THE BURGE CORRESPONDENCE BETWEEN PARTITIONS AND BINARY WORDS

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Dedicated to the memory of Ernst Straus

In 1981 Burge [3] published a very elegant solution to a problem posed by the first author, ([1], p. 156, Question 1) and described in detail below, that of finding a purely constructive bijection between two sets of partitions, each of which arises in generalization of the Rogers-Ramanujan identities. The key idea in Burge's proof is to establish separate correspondences between each of the sets of partitions and a certain set of binary words, the number being partitioned corresponding to the major index of the word.

This turned out to be a fruitful approach to partition identities, and in a subsequent article [4] Burge supplied a devastatingly simple proof of the principal result in the second author's memoir [2]. The purpose of this paper is to further strengthen the case for viewing partitions as binary words by demonstrating how this viewpoint leads to simple proofs of partition identities which on one side count partitions in which the parts are restricted to certain congruence classes. More than this, our new point of view actually leads to new such identities. As an example, we shall sketch the proof of the following generalization of the Göllnitz-Gordon identities.

THEOREM 1. *Let* δ, i, k *be integers satisfying* $\delta = 0$ *or* 1, $1 \leqq 2i - \delta < 2k$, $2i - \delta \not\equiv 2 \pmod 4$. *Given a partition of* n, f_j *denotes the number of times the part* j *appears in the partition. The following two sets of partitions are equinumerous:*

(1) *partitions of* n *in which no part is congruent to* $2 \pmod 4$ *or congruent to* $0, \pm (2i - \delta) \pmod{4k}$.

(2) *partitions of* n *in which* $f_1 \leqq 2i - 1 - \delta$, $f_j + f_{j+1} \leqq 2k - 1 - \delta$ *for all* j, f_{2j} *is always even, and if* $\delta = 0$ *and* $f_j + f_{j+1} \geqq 2k - 2$ *then*

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