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A CHARACTERIZATION OF THOSE SPACES HAVING ZERO-DIMENSIONAL REMAINDERS

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ABSTRACT. A 0-space is a completely regular Hausdorff space possessing a compactification with zero-dimensional remainder. It is well known that any rimcompact space is a 0-space, while the converse is not true. In this paper a proximal characterization of 0-spaces is presented. Those open sets U of βX for which $U \cap (\beta X \backslash X)$ is clopen in $\beta X \backslash X$ are characterized. This characterization is then utilized to define a relation α on $\mathscr{P}(X)$. It is shown that α is a proximity on X if and only if X is a 0-space. The definition of the relation α is motivated by the presentation of a proximal characterization of almost rimcompact spaces—a class of spaces intermediate between the classes of rimcompact spaces and 0-spaces.

1. Introduction and known results. The characterization of those completely regular Hausdorff spaces possessing a compactification with zero-dimensional remainder has been considered by various researchers (see for example [5], [6] and [9]). Such a compactification will be called 0-dimensional at infinity (denoted by O.I.); a 0-space is any space possessing a O.I. compactification. Recall that a space is rimcompact if it has a basis of open sets with compact boundaries ([5]). Each rimcompact space X possesses a compactification which has a basis of open sets whose boundaries are contained in X ([7], [9]). Hence a rimcompact space is a 0-space; the converse is not true ([9]). In [2] we introduced a natural generalization of rimcompactness called almost rimcompactness and obtained the following characterization, which we consider in this paper as a definition. A space X is almost rimcompact if and only if X possesses a compactification KX in which each point of $KX \backslash X$ has a basis (in KX) of open sets whose boundaries are contained in X. If KX is such a compactification of X, we say that $KX \backslash X$ is relatively 0-dimensionally embedded in KX. Hence each almost rimcompact space is a 0-space; in the same paper we show that the converse is not true. For the internal definition and a thorough discussion of almost rimcompactness, see [2] and [3].

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