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GORENSTEIN TORIC THREEFOLDS WITH ISOLATED SINGULARITIES AND CYCLIC DIVISOR CLASS GROUP

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I. Introduction. This article was motivated by the following question. Is any 19-dimensional family of $K3$ surfaces realizable as a family of divisors on a toric threefold, other than the quartics in $\mathbf{P}^3$ and the double covers of $\mathbf{P}^2$? Since toric threefolds are described so explicitly, the hope was to deduce and/or describe certain properties of these $K3$ surfaces, especially concerning their degenerations, from purely combinatorial considerations. A $K3$ surface would necessarily be an anticanonical divisor in the toric threefold, hence we require the threefold to be Gorenstein; moreover, since the general member of the family is smooth, the threefold should have only isolated singularities. Finally, by Lefschetz, the Weil Divisor Class group of the threefold injects into that of a smooth $K3$ surface, so that it should be cyclic, since the general $K3$ has cyclic Picard group.

Unfortunately this approach does not lead to 'new' descriptions of $K3$ surfaces. In this article, I will prove the following theorem.

THEOREM 1.1. *Let X be a complete Gorenstein toric threefold with isolated singularities and cyclic divisor class group. Then X is isomorphic to either*

(a) $\mathbf{P}^3$, *or*

(b) *the cone in* $\mathbf{P}^{10}$ *over the triple Veronese surface* $V \cong \mathbf{P}^2$ *embedded into* $\mathbf{P}^9$ *via cubics* $(= \mathbf{P}(1, 1, 1, 3).)$

In the first case the anticanonical divisors are the quartic $K3$ surfaces, and in the second case they are the double covers.

COROLLARY 1.2. *Let X be a projective toric threefold such that* $|-K_X|$ *contains a nonsingular $K3$ surface S with* Pic $S \cong \mathbf{Z}$. *Then* $X \cong \mathbf{P}^3$ *or* $\mathbf{P}(1, 1, 1, 3)$.

PROOF. Note that, since $S \in |-K_X|$, every $p \in S$ must be a smooth point of X. Therefore S is a Cartier divisor, so X must be Gorenstein. Since

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