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FUNCTIONS WITH PREASSIGNED LOCAL MAXIMUM POINTS

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In a recent note [1], Posey and Vaughan gave an elementary example of a continuous real valued function that has a proper local maximum at each point of a preassigned countable dense set. Let A and B be disjoint countable sets, each dense in the open interval (0, 1). We will use methods just as elementary as those used in [1] to construct a continuous nowhere differentiable function F on (0, 1) such that F has a proper local maximum at each point of A and a proper local minimum at each point of B, and has no other local maximum or minimum points.

By a triadic rational number, we mean a rational number of the form $k3^{-n}$ where n is a positive integer and k is an integer. We say that $k3^{-n}$ is even if k is even, and odd if k is odd. We begin with a lemma that is not very original.

LEMMA 1. *Let A and B be disjoint countable dense subsets of* (0, 1). *Then there is a bijective order preserving mapping g of the set of all triadic rational numbers in* (0, 1) *onto $A \cup B$ such that the odd numbers map to points in A and the even numbers map to points in B.*

PROOF. Let the sequence (a_n) be an enumeration of A and (b_n) an enumeration of B with $a_1 < b_1$. Let $g(1/3) = a_1$, $g(2/3) = b_1$. Suppose that $g(k3^{1-n})$ has been defined for some $n > 1$ and for all $k = 1, 2, \ldots, 3^{n-1} - 1$, so that g is injective and order preserving on its domain. Let $g(3^{-n})$, $g(3 \cdot 3^{-n})$, $g(5 \cdot 3^{-n})$, $\ldots$, $g((3^n - 2)3^{-n})$ be the points in $A = \{a_i\}$ with the smallest subscripts that make g still injective and order preserving. Let $g(2 \cdot 3^{-n})$, $g(4 \cdot 3^{-n})$, $g(6 \cdot 3^{-n})$, $\ldots$, $g((3^n - 1)3^{-n})$ be the points in $B = \{b_i\}$ with the smallest subscripts that make g still injective and order preserving. This completes the induction on n, and g is the required order preserving bijective mapping onto $A \cup B$.

For each $n > 0$, we define a piecewise linear function f_n on [0, 1] as follows. Let

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