MULTIPLIERS FOR SOME SPACES OF BANACH ALGEBRA VALUED FUNCTIONS

HANG-CHIN LAI

ABSTRACT. Let G be a locally compact abelian group, and A be a commutative Banach algebra. Let $C_0(G, A)$ be the Banach algebra of A-valued continuous functions on G which vanish at infinity. It is the object of this paper to characterize the space of multipliers for the space $C_0(G, A)$ regarded as a Banach algebra and regarded as an $L^1(G, A)$-module, respectively, where $L^1(G, A)$ is the Banach algebra of A-valued Bochner integrable functions on G. We prove that the space of algebra multipliers of $C_0(G, A)$ is isometrically isomorphic to $C^b(G, \mathcal{M}(A))$, the bounded continuous $\mathcal{M}(A)$-valued functions on G where $\mathcal{M}(A)$ denotes the multiplier algebra of the Banach algebra A with a bounded approximate identity. It is proved also that the $L^1(G, A)$-module homomorphisms of $C_0(G, A)$ is identified with $M(G, A)$ when A has identity of norm 1 where $M(G, A)$ is the A-valued regular Borel measure of bounded variation on G.

1. Introduction and preliminaries. Let G be a locally compact abelian group with Haar measure dt, and A be a commutative Banach algebra with a bounded approximate identity. The space $C_0(G, A)$ of A-valued continuous functions on G vanishing at infinity forms a commutative Banach algebra under pointwise products. $M(G, A)$ is the space of A-valued regular Borel measures of bounded variation on G.

For any commutative Banach algebra A, a linear map $T: A \to A$ is called a multiplier for A if $T(ab) = a(Tb) = (Ta)b$. We denote by $\mathcal{M}(A)$ the space of all multipliers for A. Clearly $\mathcal{M}(A)$ is a Banach algebra as a subalgebra of bounded linear operators on A. For the general theory of multipliers we refer to Larsen [7], and some characterizations of multipliers of Banach algebras studied also in Lai [6]. For the theory of vector valued functions or vector measures, one can consult Dinculeanu [1], [2] and Johnson [4] for the spaces of Banach algebra valued functions on a locally compact group.