ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 15, Number 1, Winter 1985

SOME DISTORTION THEOREMS FOR A CLASS OF CONVEX FUNCTIONS

RICHARD FOURNIER

1. Introduction. Let A denote the class of analytic functions f in the unit disc $E = \{z \mid |z| < 1\}$ with $f(0) = f'(0) - 1 = 0$. For a function $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ in A, Ruscheweyh has defined [4] the δ-neighbourhood of f as

$$N_\delta(f) = \{g(z) = z + \sum_{k=2}^{\infty} b_k z^k \,\Big|\, \sum_{k=2}^{\infty} k\,|a_k - b_k| \leqq \delta\}.$$

This paper deals with the following subclasses of A.

$$T = \{f \in A \mid \left|\frac{zf'(z)}{f(z)} - 1\right| < 1,\ z \in E\}$$

$$\tilde{T} = \{f \in A \mid \left|\frac{zf''(z)}{f'(z)}\right| < 1,\ z \in E\}.$$

The functions in $\tilde{T}(T)$ are convex (starlike) univalent functions. The following result was proved in [1].

THEOREM A. *Let $g \in \tilde{T}$. Then $N_\delta(g) \subset T$ for $\delta = 1/e$. Moreover if for a function $g \in T$ we have* $\sup_{z \in E} |(zg'(z)/g(z)) - 1| = 1$, *then $N_\delta(g) \not\subset T$ for any $\delta > 0$.*

It follows clearly from Theorem A and the compacity of the class $\tilde{T}$ that

$$\sup_{\substack{|z|<1 \\ g \in \tilde{T}}} \left|\frac{zg'(z)}{g(z)} - 1\right| =: \rho < 1$$

and therefore we have $\tilde{T} \subset T$.

In this paper we will be mainly concerned with the precise determination of ρ. Some new distortion theorems for the classes T and $\tilde{T}$ will also be obtained.

2. An estimate for ρ. It is easily seen from the definitions that

Received by the editors on June 14, 1983.
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