ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 14, Number 4, Fall 1984

AN INTRODUCTION TO LOCALLY SEMIALGEBRAIC SPACES

HANS DELFS AND MANFRED KNEBUSCH

Dedicated to the memory of Gus Efroymson

In our paper [**4**] we introduced the category of semialgebraic spaces over an arbitrary real closed field R. This category seems to be the natural framework in which to describe "topological" phenomena over R. Nevertheless it turns out to be too small for some purposes. For example, every semialgebraic covering $p\colon M \to N$ of a semialgebraic space N (defined in the obvious way) necessarily has finite degree since its fibres are zero-dimensional semialgebraic spaces and thus are finite sets. One also wants to consider "covering spaces of infinite degree over N. These spaces should, at least locally, look like semialgebraic spaces.

So we are forced to enlarge the category of semialgebraic spaces and to introduce locally semialgebraic spaces. In the first two sections of this paper we give a review of the basic definitions and properties of locally semialgebraic spaces and maps and illustrate this new concept by examples. In §3 we consider locally finite simplicial complexes. They are the most important examples of locally semialgebraic spaces since a large class of locally semialgebraic spaces, namely all paracompact and regular spaces, can be triangulated. In §5 we discuss the coverings of a space. These coverings were the initial reason for developing the whole theory. As in topology they are classified by the subgroups of the semialgebraic fundamental group defined in §4.

Since this paper is meant to be a survey, we have omitted most proofs. We are planning a more detailed treatment of the whole subject in the near future. Complete proofs of all statements in this paper will appear at this time.

Throughout the paper R denotes an arbitrary real closed field.

1. Definition of locally semialgebraic spaces and examples. Locally semialgebraic spaces are ringed spaces which locally look like semialgebraic spaces. In order to explain and interpret this in detail we start with the following ad hoc definition.

DEFINITION 1. A generalized topological space is a set M together with a