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TOPOLOGY AND REAL ALGEBRAIC GEOMETRY

SELMAN AKBULUT

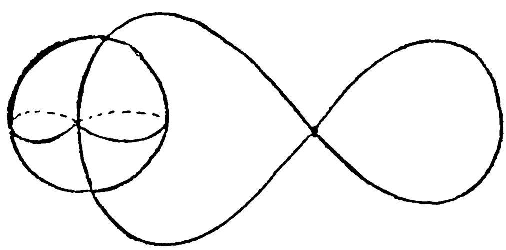
Dedicated to the Memory of Gus Efroymson

The problem of solving real polynomial equations long fascinated people. Starting with a finite number of points on the real line one can get many complicated spaces as zero sets of polynomials. For example every closed P.L. manifold occurs as an algebraic set [3]. The main goal of understanding the topology of algebraic sets is to topologically characterize the image of the forgetful functor

$$\{\text{Algebraic sets}\} \to \{\text{Topological spaces}\}.$$

Rather than discussing the history and development of this problem we simply refer the reader to the surveys ([5], [7]) and instead describe the program which I developed jointly with H. King towards solving this problem.

Lets take an algebraic set X in $\mathbf{R}^3$ given by $(2x^2 + 2y^2 - 1)((x - 1)^4 - (x - 1)^2 + y^2)^2 + 2z^2 = 0$ X looks like this.



By resolving singularities of X or just by inspecting, we see that X is obtained by a disjoint union of smooth manifolds by doing some identifications. More specifically, $X = \bigcup_{i=0}^{2} V_i/\sim$ where V_0, V_1, V_2 are a point, a circle, and a 2-sphere respectively; $\sim$ indicates that the equator of V_2 is folded onto V_1, and the two points of V_1 are folded onto V_0 (i.e., identified).

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