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A POLYNOMIAL-TIME ALGORITHM FOR THE TOPOLOGICAL TYPE OF A REAL ALGEBRAIC CURVE —EXTENDED ABSTRACT

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Dedicated to the memory of Gus Efroymson

1. Introduction. Let $f(x, y, z)$ be a homogeneous polynomial with rational coefficients. Let C_f be the real projective curve defined by $f = 0$. It is well known [9] that if C_f is nonsingular, then it is a compact one-dimensional manifold, and so homeomorphic to a disjoint union of circles. A circle can have either a one-sided or two-sided imbedding in $\mathbf{R}P^2$; in the latter case it has both an interior (homeomorphic to a disk), and an exterior (homeomorphic to a Möbius strip). The two-sided components of C_f are called ovals. If f has even degree, then every component of C_f is an oval; if degree (f) is odd, every component except one is an oval.

Curves C_1 and C_2 have the same topological type if there is a homeomorphism φ: $\mathbf{R}P^2 \rightarrow \mathbf{R}P^2$ which maps C_1 onto C_2. Each oval of a nonsingular curve C_f is either inside or outside any other; the partial ordering of the ovals induced by this inclusion relation, together with the parity of the degree of f, determine the topological type of the curve.

We present an algorithm which, given $f(x, y, z)$ with rational coefficients, determines whether C_f is nonsingular, and if so, determines the ordering of its ovals.

2. Description of algorithm. We may assume that f is squarefree; (if not, we can replace f by its greatest squarefree divisor h, as $C_f = C_h$). The main step of the algorithm is construction of a cellular decomposition D_f of $\mathbf{R}P^2$ such that every component of C_f is a union of cells of D_f. The following description of D_f is produced: (1) a list of the pairs of adjacent cells (two cells are adjacent if their union is connected), and (2) a list of the cells contained in C_f. In the course of constructing D_f we determine if C_f has singularities, and if so, halt.

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