ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 14, Number 4, Fall 1984

SURVEY ON THE TOPOLOGY OF REAL ALGEBRAIC SETS

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Dedicated to the memory of Gus Efroymson

This is a survey of some aspects of the topology of real algebraic sets. It reflects my personal view of the subject. I have been interested in other things lately and have not kept up with various colleague's work so I hope relevant additional information will be brought up in the discussion. The main point I would like to make is that there is much interesting mathematics to be done here. Furthermore, it can be approached at an elementary level. I am particularly gratified with the interest in representing $\mathbf{Z}/2\mathbf{Z}$ homology classes by algebraic subsets.

The first things we will discuss are restrictions on the topology of real algebraic sets. The first restriction is that a real algebraic set is triangulable. The early proofs of this that I know of were incorrect. I do not know of a correct proof before Lojasiewicz, **[14]** but perhaps there were earlier correct proofs. A very nice proof was given by Hironaka **[11]**.

Sullivan found the next restriction in **[17]**. If V is a real algebraic set and $\rho \in V$ then triangulability of V implies that ρ has a neighborhood homeorphic to the cone on some space X. Sullivan's result is that X must have even Euler characteristic. He gets this result by looking at the complexification $V_{\mathbf{C}}$ of V and the involution on $V_{\mathbf{C}}$ induced by complex conjugation.

The next restriction was found by Akbulut and King in **[2]**. If V is a real algebraic set, then a simple construction shows that the one point compactification of V is a real algebraic set also. This shows for example that V is the union of a compact set and set homeomorphic to $X \times [0, 1)$ for some compact polyhedron X with even Euler characteristic.

The final restriction was also found by Akbulut and King in **[4]**. It is complicated to describe but it comes from a synthesis of Hironaka's resolution of singularities of algebraic varieties **[10]**, Sullivan's resolution of singularities of topological spaces **[18]**, and Akbulut and King's notion of A-spaces **[3]**.

One possibility for obtaining more restrictions on the topology of real algebraic sets is to look more carefully at the complexification. A compact