ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 14, Number 4, Fall 1984

RESEARCH ANNOUNCEMENT ON EXTENDING NASH FUNCTIONS OFF SINGULAR CURVES

GUSTAVE EFROYMSON

The general extension problem is: given sets $W \subset V$ and a function f: $W \to \mathbf{R}$, to find a function g: $V \to \mathbf{R}$ with $g = f$ on V. We say g extends f. To make the problem interesting, we need restrictions on f and g. In our case we want f and g to be Nash. The most general extension theorem about Nash functions doesn't quite fit the above description.

THEOREM [2] *Let V be a Nash variety in $\mathbf{R}^n$ (i.e., $V = h^{-1}(0)$ for a Nash function h: $\mathbf{R}^n \to \mathbf{R}$). Suppose U is an open neighborhood of V and f a Nash function f: $U \to \mathbf{R}$. Then there exists a Nash function g defined on $\mathbf{R}^n$ with $g = f$ on V.*

Note we have to assume f is defined on a neighborhood of V to extend it. But from the above theorem it easily follows that

THEOREM. [2] *If V is a non-singular variety in $\mathbf{R}^n$ and f: $V \to \mathbf{R}$ is Nash then there exists g: $\mathbf{R}^n \to \mathbf{R}$ extending f.*

At this point, it would be a good idea to say what we mean by a Nash function on a possibly singular variety. Recall first that if V is nonsingular, every point of V has a neighborhood which is essentially euclidean, and so the usual definition applies, i.e., f on V is Nash if f is analytic on V and f is algebraic. For a singular point of V, I don't know what an analytic function is, so I will define a globally algebraic function on V to be Nash at a point p of V if f has an analytic extension to some neighborhood of P in $\mathbf{R}^n$.

QUESTION. Can you always extend a Nash function f: $V \to \mathbf{R}$ to g: $\mathbf{R}^n \to \mathbf{R}$ where V is a possibly singular variety?

ANSWER. No. For example there is a Nash function on the Whitney umbrella $(x^2 + y^2)z = x^3$, which can't be extended.

I think the problem with the Whitney umbrella is that it is not coherent [4]. Since curves are always coherent, they are a good starting place for proving an extension result.