

IRREDUCIBLE REPRESENTATIONS OF INSEPARABLE C^* -ALGEBRAS

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ABSTRACT. Irreducible representations of a C^* -algebra are shown to induce irreducible representations on certain arbitrarily large separable subalgebras. Many structural properties of C^* -algebras can also be reduced to separable subalgebras.

1. Introduction. There are several problems concerning C^* -algebras which were solved some time ago assuming separability conditions, but which are outstanding in inseparable cases. For example, Dixmier [3] showed in 1960 that prime ideals of separable C^* -algebras are primitive, but it is unknown whether this is valid in inseparable cases. Meanwhile, Glimm [6] showed that the following conditions on a separable C^* -algebra A are equivalent:

- (i) A is postliminal,
- (ii) A is smooth,
- (iii) A has no (factorial) representations of type III,
- (iv) A has no (factorial) representations of type II,
- (v) Irreducible representations of A with equal kernels are unitarily equivalent.

Subsequently, Sakai [7, 8] established the equivalence of conditions (i), (ii) and (iii) even for inseparable C^* -algebras, but it remains unknown whether they are implied by (iv) or by (v) (see [2, 12] for some partial results about (iv)).

It was already implicit in Glimm's argument that any antiliminal C^* -algebra A contains a separable C^* -subalgebra B which is not postliminal. Passing to an antiliminal quotient, the same result follows whenever A is not postliminal. This paper introduces a technique for reducing some properties of this type to the corresponding properties of the separable subalgebras. For example, it will be shown that (prime) antiliminal C^* -algebras contain arbitrarily large separable (prime) antiliminal subalgebras.

Throughout the paper, suffixes i, j, m, n, r , etc. take on (non-negative) integer values.

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