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THE CENTRALIZER OF THE LAGUERRE POLYNOMIAL SET

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1. Introduction. By a polynomial set (p.s.) we mean a sequence $P = \{P_0(x), P_1(x), P_2(x), \cdots\}$ of polynomials in which $P_0(x) \neq 0$ and $P_n(x)$ is of exact degree n. In this work we shall be interested in sets (or classes) whose elements are themselves polynomial sets. This point of view is not new. Appell [2] considered the class $\mathscr{A}$ of Appell polynomials $A = \{A_n(x)\}$ whose generating function is

$$A(t)e^{xt} = \sum_{n=0}^{\infty} A_n(x)\frac{t^n}{n!}. \tag{1.1}$$

The Sheffer class $\mathscr{S}$ [6] is the class of all p.s. $S = \{S_n(x)\}$ for which

$$A(t)e^{xH(t)} = \sum_{n=0}^{\infty} S_n(x)\frac{t^n}{n!}. \tag{1.2}$$

Similarly the Boas-Buck class $\mathscr{B}$ consists of all p.s. B for which [3]

$$A(t)\Phi(xH(t)) = \sum_{n=0}^{\infty} \phi_n B_n(x)t^n, \tag{1.3}$$

where in these formulas $A(t)$, $H(t)$ and $\Phi(t)$ are formal power series such that $A(0) \neq 0$, $H(0) = 0$ but $H'(0) \neq 0$, and $\Phi(t) = \phi_0 + \phi_1 t + \phi_2 t^2 + \cdots$ with $\phi_k \neq 0$ for all $k \geqq 0$. (1.1) is obtained when $H(t) = t$ and $\Phi(t) = e^t$.

Many of the well known p.s. are included in one or more of the above classes. For example, the Hermite p.s. is in $\mathscr{A}$ as well as in $\mathscr{S}$. The Laguerre p.s. $L^{(\alpha)}$ is in $\mathscr{S}$. Other examples are the Abel, the Meixner, the Bernoulli, and the Boole polynomial sets.

Appell [2], Sheffer [6] as well as Rota, Kahaner and Odlysko [5] (see also [4]) gave sets of polynomials ($\mathscr{A}$ in [2], $\mathscr{S}$ in [4], [5], [6]) an algebraic structure by defining multiplication in the following manner.

Let $P = \{P_n(x)\}$ and $Q = \{Q_n(x)\}$ be two elements of the set under consideration. Let, furthermore, $P_n(x) = \sum_{k=0}^{n} p_{nk}x^k$ and $Q_n(x) =$

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