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DIFFERENTIAL EQUATIONS ON LIE GROUPS AND TORI THE WAVE EQUATIONS AND HUYGENS' PRINCIPLE

H. D. FEGAN

1. Introduction. The purpose of this note is to describe one way in which a differential equation on a compact Lie group can be transferred to another equation on the maximal torus. This procedure is then illustrated by applying it to the case of the wave equation. From this we obtain a well known result about the non-existence of Huygens' principle on a compact Lie group. More interestingly we obtain the existence of Huygens' principle for a shifted wave equation.

Let G be a compact Lie group. Let ρ be half the sum of the positive roots for the Lie algebra t of some maximal torus $T \subseteq G$. (See Section 2). Let $\|\rho\|$ denote the norm of ρ with respect to an appropriate norm on t^*. (The norm $\| \quad \|$ is given by the killing form if G is semisimple. Again we refer to Section 2 for details.) The result on the existence of Huygen's principle is as follows.

THEOREM 1.1. *Let G be odd dimensional. Then the shifted wave equation*

$$(\Delta + \|\rho\|^2)u = \partial^2 u/\partial t^2 \tag{1.1}$$

satisfies Huygens' principle.

We explain below precisely what Huygen's Principle involves.

Notice that since $\|\rho\|^2$ is a constant, see Section 2, the Laplacian is shifted by a constant for Huygens' principle to hold. This result is particularly interesting in the light of the results in [1] and [2]. In [1] the asymptotic expansion for the trace of the heat equation is given. If we take the heat equation as $\Delta u + \partial u/\partial t = 0$ the expansion is, for a suitable constant c.

$$Z(t) \approx ct^{-k/2}(1 + \|\rho\|^2 + \cdots) \text{ as } t \to 0. \tag{1.2}$$

On the other hand if we use the shifted Laplacian and take the heat equation as $(\Delta + \|\rho\|^2)u + \partial u/\partial t = 0$, the expansion becomes

$$Z(t) \approx ct^{-k/2} \text{ as } t \to 0. \tag{1.3}$$

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