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ZEROS OF OPERATORS ON FUNCTIONS AND THEIR ANALYTIC CHARACTER

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1. Introduction. This is a survey of recent work on the behavior of the zeros of iterated operators on analytic functions. It is motivated by the classical survey paper of G. Pólya **[50]**, the more recent survey due to Boas **[8]** and the collected papers by Pólya **[49]**. Some old and perhaps forgotten results coupled with new and interesting developments have helped to add a new dimension to some classical results as well as to some as yet unsolved problems. The classical problems include the determination of the behavior of the zeros of the successive derivatives of an analytic function and the influence of the behavior of the sign changes of the derivatives of a function on its analytic character.

The present paper is by no means exhaustive. The author's intention is merely to present recent results designed to indicate the flavor of the work being done on the topics under discussion. The reader should consult the original papers for more detailed information and perhaps will tackle some of the many interesting problems that remain.

2. The complex domain; the case of derivatives. The classical problem involves the determination of the distribution of the zeros of the successive derivatives $f(z)$, $f'(z)$, $f''(z)$, $\ldots$. To be more precise, following Pólya, given a function $f(z)$ analytic on a domain D, we say that a point z_0 lies in the final set S of f when every neighborhood of z_0 contains zeros of infinitely many of the derivatives of f. The final set thus determines the final location of the zeros of the successive derivatives.

3. Meromorphic Functions. For meromorphic functions, the final set for the zeros of the successive derivatives is easy to describe. Pólya's remarkable theorem **[52]** says that the poles behave like repellers for the zeros of the successive derivatives. Specifically, if a meromorphic function f has at least two distinct poles, then a point z lies in the final set S of f if and only if z is equidistant from the two poles that are nearest to it. If f has only one pole, the final set of f is empty. Proofs of this theorem can be found in Whittaker **[82]** and Hayman **[30]**. When f is meromorphic

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