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ON THE IMPRIMITIVITY THEOREM FOR ALGEBRAIC GROUPS

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Let G be an affine algebraic group, defined over the algebraically closed field k, and let A be a commutative k-algebra which is a rational G-module such that G acts on A as k-algebra automorphisms. An $A.G$-module M is an A module and a rational G-module such that $g(am) = g(a)g(m)$ for $g \in G$, $a \in A$ and $m \in M$, and a morphism of $A.G$-modules is a G-linear A-homomorphism. The $A.G$-modules, and their morphisms, form an abelian category which we denote Mod$(A.G)$.

In [7, Theorem 3.1, p. 42] Parshall and Scott prove that if H is an affine algebraic subgroup of G such that the homogeneous space G/H is affine, then Mod$(k[G/H].\ G)$ is equivalent to Mod(H), the category of rational H-modules. The proof uses their earlier theorem [2, Theorem 4.3, p. 9] that G/H being affine implies that the induction functor from Mod(H) to Mod(G) is exact. See also [9].

The point of the present note is to observe that a slightly more general version of the above category equivalence can be derived directly from the fundamental (and easily proven) algebraic fact that if A is a simple $A.G$-module, then $A.G$-modules are all A-flat, due to I. Dorai swamy [3, Cor. 2.3, p. 792]. In the version presented here, it is the inverse of the induction functor that is easier to consider. The theorem is preceded by some standard observations on Hopf algebras and followed by some applications. The notation already introduced is retained throughout.

To define the functor, we assume there is a k-algebra homomorphism $\alpha\colon A \to k$. Let Y be the affine scheme represented by A, on which the group scheme G represented by $k[G]$ acts. Then the functor which assigns to each commutative k-algebra C the stabilizer in $G(C)$ of the α of $Y(C)$ is also an affine group scheme: the fibre product $Gx_Y\{e\}$ where the right map $\{e\} \to Y$ is $e \to \alpha$ and the left map $G \to Y$ is $g \to g\alpha$. It follows that the algebra $B = k[G] \otimes_Y k$ representing $Gx_y\{e\}$ is a Hopf algebra and that $k[G] \to B$ is a Hopf algebra morphism whose kernel J is a Hopf ideal. The range of the functor will be the category of B comodules.

We need to recall how G-modules can be regarded as $k[G]$-comodules. If M is a rational G-module, the map $\gamma_M\colon M \to M \otimes k[G]$, defined by

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