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CONTINUOUS FUNCTIONS AND MINIMAL REGULARITY

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1. Introduction. A T_3 space X is said to be R-closed if it is a closed subspace of every T_3 space in which it is embedded, minimal regular if no T_3 topology on the underlying set is strictly weaker than the given topology, and strongly minimal regular if there is a base $\mathscr{V}$ for the topology of X such that each member of $\mathscr{V}$ is the complement of an R-closed subspace of X. It is well known that every compact Hansdorff space is strongly minimal regular, every strongly minimal regular space is minimal regular, and every minimal regular space is R-closed. None of these implications is reversible. (See [**1**, pp. 104–105], [**10**, p. 297], [**8**]).

Every T_3 space that is the continuous image of an R-closed space is itself R-closed [**1**, Th. 4.16(b)]. In [**2**], M. P. Berrizabal and C. F. Blakemore showed that minimal regularity is not in general preserved by continuous functions onto T_3 spaces. Neither is strong minimal regular, for the domain space of the example in [**2**] is not only minimal regular but strongly minimal regular. In [**7**], it was shown that the class of R-closed spaces coincides with the class of T_3 continuous images of strongly minimal regular spaces.

In this paper, we characterize in terms of filterbases the T_3 spaces all of whose T_3 continuous images are minimal regular and the T_3 spaces all of whose T_3 continuous images are strongly minimal regular. We also describe a noncompact T_3 space whose every T_3 continuous image is strongly minimal regular. This same space has another interesting property, i.e., there is a decreasing chain of nonempty, strongly minimal regular subspaces such that the intersection of these subspaces is empty.

2. Preliminaries. Many of the properties we deal with in this paper have convenient characterizations in terms of filterbases.

For a filterbase $\mathscr{B}$ on topological space S, we let $\mathrm{ad}_S\mathscr{B}$ denote the adherence of $\mathscr{B}$ with respect to S (i.e., the set of all cluster points of $\mathscr{B}$ in S). When no confusion is likely to arise, we may simply write ad $\mathscr{B}$.

Let f be a function from a topological space S onto a topological space T. If $\mathscr{B}$ is a filterbase on S, then $f(\mathscr{B})$ will denote the filterbase $\{f(B) | B \in \mathscr{B}\}$

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